DIFFERENTIATION

1 Find an equation for the tangent to the curve with equation $y = x^2 + \ln(4x - 1)$ at the point on the curve where $x = \frac{1}{2}$.

2 A curve has the equation $y = \sqrt{8 - e^{2x}}$.

The point P on the curve has y-coordinate 2.

a Find the x-coordinate of P.

b Show that the tangent to the curve at P has equation

$$2x + y = 2 + \ln 4.$$

3 A curve has the equation $y = 2x + 1 + \ln(4 - 2x)$, $x < 2$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve.

c Determine the nature of this stationary point.

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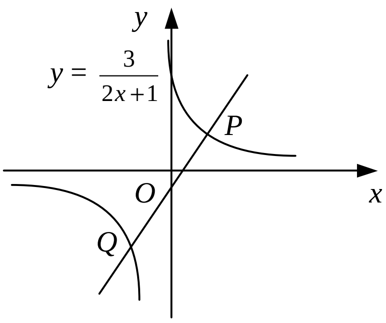


The diagram shows the curve with equation $y = \frac{3}{2x+1}$.

a Find an equation for the normal to the curve at the point P (1, 1).

The normal to the curve at P intersects the curve again at the point Q.

b Find the exact coordinates of Q.

5 A quantity N is increasing such that at time t seconds,

$$N = ae^{kt}.$$

Given that at time $t = 0$, $N = 20$ and that at time $t = 8$, $N = 60$, find

a the values of the constants a and k,

b the value of N when $t = 12$,

c the rate at which N is increasing when $t = 12$.

6 $$f(x) \equiv (5 - 2x^2)^3.$$

a Find $f'(x)$.

b Find the coordinates of the stationary points of the curve $y = f(x)$.

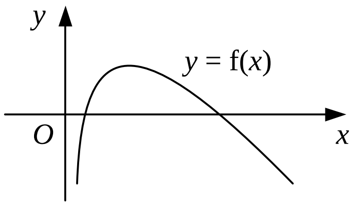
c Find the equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate $\frac{3}{2}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

7 A curve has the equation $y = 4x - \frac{1}{2}e^{2x}$.

a Find the coordinates of the stationary point of the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

8



The diagram shows the curve $y = f(x)$ where $f(x) = 3 \ln 5x - 2x$, $x > 0$.

a Find $f'(x)$.

b Find the x-coordinate of the point on the curve at which the gradient of the normal to the curve is $-\frac{1}{4}$.

c Find the coordinates of the maximum turning point of the curve.

d Write down the set of values of x for which $f(x)$ is a decreasing function.

9 The curve C has the equation $y = \sqrt{x^2 + 3}$.

a Find an equation for the tangent to C at the point A $(-1, 2)$.

b Find an equation for the normal to C at the point B $(1, 2)$.

c Find the x-coordinate of the point where the tangent to C at A meets the normal to C at B.

10 A bucket of hot water is placed outside and allowed to cool. The surface temperature of the water, T °C, after t minutes is given by

$$T = 20 + 60e^{-kt},$$

where k is a positive constant.

a State the initial surface temperature of the water.

b State, with a reason, the air temperature around the bucket.

Given that $T = 30$ when $t = 25$,

c find the value of k,

d find the rate at which the surface temperature of the water is decreasing when $t = 40$.

11 $$f(x) \equiv x^2 - 7x + 4 \ln \left(\frac{x}{2}\right), \quad x > 0.$$

a Solve the equation $f'(x) = 0$, giving your answers correct to 2 decimal places.

b Find an equation for the tangent to the curve $y = f(x)$ at the point on the curve where $x = 2$.

12 A curve has the equation $y = x^2 - \frac{8}{x-1}$.

a Show that the x-coordinate of any stationary point of the curve satisfies the equation

$$x^3 - 2x^2 + x + 4 = 0.$$

b Hence, show that the curve has exactly one stationary point and find its coordinates.

c Determine the nature of this stationary point.